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Mathematical Reasoning

Introduction

Logic is the science of reasoning.

By reasoning we draw valid inferences from some well-known facts.

In mathematical language, there are two kinds of reasoning, namely *inductive* and *deductive.*

Mathematical induction is some sort of inductive reasoning, which we have studied in an earlier chapter.

Now, in this chapter we shall discuss deductive reasoning.

SENTENCES *A group of words which makes complete sense is called a sentence.*

We communicate our ideas to others through sentences.

Sentences are classified as *declarative* (or *assertive*); *imperative; exclamatory* and *interrogative*.

DECLARATIVE SENTENCES *A sentence that makes an assertion is called a declarative or assertive sentence.*

EXAMPLES *Each of the following is a declarative sentence.*

(i) The earth revolves round the sun.

(ii) The sum of the angles of a triangle is 180°.

(iii) 2 *is the only even prime number.*

IMPERATIVE SENTENCES *A sentence that expresses a request or a command is called an imperative sentence.*

EXAMPLES *Each of the following is an imperative sentence.*

(i) Switch on the light.

(ii) Do your homework.

(iii) Give me a glass of water.

(iv) Keep quiet.

INTERROGATIVE SENTENCES *A sentence that asks some question is called an interrogative sentence.*

EXAMPLES *Each of the following is an interrogative sentence.*

(i) Where are you coming from?

(ii) How far is Mumbai from here?

(iii) Is every set finite?

EXCLAMATORY SENTENCES *A sentence that expresses some strong feeling is called an exclamatory sentence.*

EXAMPLES *Each of the following is an exclamatory sentence.*

(i) What a beautiful girl!

(ii) Listen to me, Tanvy!

(iii) May you live long!

(iv) Hurrah! We have won.

Actually, in mathematical terms, we shall be interested only in particular types of declarative sentences, called statements.

STATEMENTS *A declarative sentence which can be judged to be true or false but not both, is called a mathematically acceptable statement or simply a statement.*

EXAMPLE 1 *Check whether the following sentences are statements. Give reasons for your answer.*

(i) The earth is a planet.

(ii) There are 35 days in a month.

(iii) Mathematics is difficult.

(iv) Every square is a rectangle.

(v) Every set is a finite set.

(vi) Every real number is a complex number.

(vii) 9 is less than 7.

(viii) The square of a natural number is an even number.

SOLUTION (i) It is a scientifically established fact that the earth is a planet. So, the given sentence is true.

Hence, it is a statement.

(ii) We know that the maximum number of days in a given month is 31. So, the given sentence is false.

Therefore, it is a statement.

(iii) For some people mathematics can be easy and for some others it can be difficult.

So, we cannot judge whether the given sentence is true or false.

Hence, it is not a statement.

(iv) In a square, every angle measures $90°$ and its opposite sides are equal. So, it is a rectangle.

Thus, the given sentence is true.

Hence, it is a statement.

(v) Since there are sets which are not finite, so the given sentence is false.

Therefore, it is a statement.

(vi) Every real number a can be written as $a = (a + 0i)$.

So, every real number is a complex number.

Therefore, the given sentence is true.

Hence, it is a statement.

(vii) Since 9 is greater than 7, so the given sentence is false.
Hence, it is a statement.

(viii) We know that 2 and 3 are natural numbers such that $2^2 = 4$, which is even and $3^2 = 9$, which is odd.
So, the given sentence is sometimes true and sometimes false.
Therefore, it is not a statement.

REMARK (I) *Sentences involving variable time such as 'yesterday', 'today' and 'tomorrow' are not statements.*

EXAMPLES *None of the sentences given below is a statement.*
(i) Today is Sunday.
(ii) Yesterday was a windy day.
(iii) Tomorrow is a holiday.

REMARK (II) *Sentences involving variable places such as 'here', 'there', etc., are not statements.*

EXAMPLES *None of the sentences given below is a statemement.*
(i) Meerut is far from here.
(ii) This place is very hot.
(iii) Rohit will reach there in time.

REMARK (III) *Sentences involving variable pronouns such as 'he', 'she', etc., are not statements.*

EXAMPLES *None of the sentences given below is a statemement.*
(i) He is a doctor.
(ii) She is a graduate.

EXAMPLE 2 *Check whether the following sentences are statements.*
(i) Answer this question.
(ii) Everyone in this room is rich.
(iii) There is no rain without clouds.
(iv) Mathematics is fun.
(v) She is a commerce graduate.
(vi) $\sqrt{2}$ is a rational number.
(vii) Fire is always hot.
(viii) The sides of a quadrilateral have equal length.

SOLUTION (i) It is an order and therefore, it is not a statement.

(ii) From the context it is not clear which room is referred here and also the term rich is not well defined.
So, the given sentence is not a statement.

(iii) This sentence is always true and therefore, it is a statement.

(iv) For those who like mathematics it may be a fun but for others it may not be.

So, the given sentence cannot be judged to be true or false.

Hence, it is not a statement.

(v) It is not known who she is. So, the given sentence is not a statement.

(vi) Clearly, $\sqrt{2}$ is an irrational number. So, the given sentence is false.

Therefore, it is a statement.

(vii) Clearly, the given sentence is always true.

Hence, it is a statement.

(viii) All the sides of a square as well as that of a rhombus are equal. But, the sides of a trapezium as well as that of a rectangle are inequal.

So, the given sentence is sometimes true and sometimes false.

Hence, it is not a statement.

NEGATION OF A STATEMENT

The denial of an assertion contained in a statement is called its negation.

~ p denotes the negation or denial of p.

The denial of a statement can be expressed in various ways.

EXAMPLE 3 *Let p: Kolkata is a city. Express the denial of p in three different ways.*

SOLUTION This can be done as shown below.

~ *p*: Kolkata is not a city.

or ~ *p*: It is false that Kolkata is a city.

or ~ *p*: It is not the case that Kolkata is a city.

EXAMPLE 4 *Write the negation of each of the following statements in two different ways.*

(i) Africa is a continent.

(ii) $\sqrt{5}$ *is rational.*

(iii) All integers are rational numbers.

(iv) Some prime numbers are odd numbers.

(v) Everyone in Germany speaks German.

SOLUTION Given below are the required negations:

(i) Africa is not a continent.

Or, It is false that Africa is a continent.

(ii) $\sqrt{5}$ is not rational.

Or, It is false that $\sqrt{5}$ is rational.

(iii) There exists an integer which is not a rational number.

Or, It is false that all integers are rational numbers.

(iv) No prime number is an odd number.

Or, It is false that some prime numbers are odd numbers.

(v) At least one person in Germany does not speak German.

Or, It is false that everyone in Germany speaks German.

EXAMPLE 5 *Write the negation of each of the following statements:*

p: All birds have wings.

q: For every real number x, $x^2 > x$.

r: There exists a real number x *such that* $0 < x < 1$.

s: For every real number x, *either* $x > 1$ *or* $x < 1$.

SOLUTION We have:

~ p: There exists a bird which has no wings.

~ q: There exists a real number x such that x^2 is not greater than x.

~ r: There does not exist a real number x such that $0 < x < 1$.

~ s: There exists a real number x such that neither $x > 1$ nor $x < 1$.

EXERCISE 29A

1. Which of the following sentences are statements? In case of a statement mention whether it is true or false.

(i) The sun is a star.

(ii) $\sqrt{7}$ is an irrational number.

(iii) The sum of 5 and 6 is less than 10.

(iv) Go to your class.

(v) Ice is always cold.

(vi) Have you ever seen the Red Fort?

(vii) Every relation is a function.

(viii) The sum of any two sides of a triangle is always greater than the third side.

(ix) May God bless you!

2. Which of the following sentences are statements? In case of a statement, mention whether it is true or false.

(i) Paris is in France.

(ii) Each prime number has exactly two factors.

(iii) The equation $x^2 + 5|x| + 6 = 0$ has no real roots.

(iv) $(2 + \sqrt{3})$ is a complex number.

(v) Is 6 a positive integer?

(vi) The product of –3 and –2 is –6.

(vii) The angles opposite to the equal sides of an isosceles triangle are equal.

(viii) Oh! it is too hot.

(ix) Monika is a beautiful girl.

(x) Every quadratic equation has at least one real root.

3. Which of the following statements are true and which are false? In each case give a valid reason for your answer.

(i) p: $\sqrt{11}$ is an irrational number.

(ii) q: Circle is a particular case of an ellipse.

(iii) r: Each radius of a circle is a chord of the circle.

(iv) s: The centre of a circle bisects each chord of the circle.

(v) t: If a and b are integers such that $a < b$, then $-a > -b$.

(vi) u: The quadratic equation $x^2 + x + 1 = 0$ has no real roots.

4. Write the negation of each of the following statements:

(i) Every natural number is greater than 0.

(ii) Both the diagonals of a rectangle are equal.

(iii) The sum of 4 and 5 is 8.

(iv) The number 6 is greater than 4.

(v) Every natural number is an integer.

(vi) The number –5 is a rational number.

(vii) All cats scratch.

(viii) There exists a rational number x such that $x^2 = 3$.

(ix) All students study mathematics at the elementary level.

(x) Every student has paid the fees.

(xi) There is some integer k for which $2k = 6$.

(xii) None of the students of this class has passed.

ANSWERS (EXERCISE 29A)

1. (i) Statement, True (ii) Statement, True
(iii) Statement, False (iv) Imperative sentence, not a statement
(v) Statement, True (vi) Interrogative sentence, not a statement
(vii) Statement, False (viii) Statement, True
(ix) Exclamatory sentence, not a statement

2. (i) Statement, True (ii) Statement, True
(iii) Statement, True (iv) Statement, False
(v) Interrogative sentence, not a statement
(vi) Statement, False (vii) Statement, True
(viii) Exclamatory sentence, not a statement
(ix) Not a statement, the term 'beautiful' is vague.
(x) Statement, False

3. (i) True, as 11 being prime $\sqrt{11}$ is irrational.

(ii) True. The equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Putting $b = a$, it becomes $x^2 + y^2 = a^2$, which is the equation of a circle.

(iii) False, since the radius of a circle is not a chord.

(iv) False, since there are chords which do not pass through the centre

(v) True, since $a < b \Rightarrow -a > -b$ is true.

(vi) True. Since $D = (1^2 - 4 \times 1 \times 1) - 3 < 0$.

4. (i) There exists a natural number which is not greater than 0.

(ii) It is false that both the diagonals of a rectangle are equal.

(iii) It is false that the sum of 4 and 5 is 8.

(iv) The number 6 is not greater than 4.

(v) There exists a natural number which is not an integer.

(vi) The number –5 is not a rational number.

(vii) There exists a cat which does not scratch.

(viii) There does not exist a rational number x such that $x^2 = 3$.

(ix) There exists a student who does not study mathematics at the elementary level.

(x) At least one student has not paid the fees.

(xi) For all integers k, $2k \neq 6$.

(xii) At least one student of this class has passed.

LOGICAL CONNECTIVES

SIMPLE STATEMENTS *A statement which does not contain any other statement as its component part is called a simple statement.*

EXAMPLES *Each of the following is a simple statement.*

(i) Chennai is the capital of Tamil Nadu.

(ii) The set of real numbers is an infinite set.

(iii) The sum of 6 *and* 3 *is greater than* 10.

COMPOUND STATEMENTS *A compound statement is a combination of two or more simple sentences.*

The method of combining two or more simple sentences is known as *compounding*.

LOGICAL CONNECTIVES *The words or phrases which connect two or more simple sentences are called logical connectives or simply connectives.*

These connectives are:

(i) ***and*** (ii) ***or*** (iii) ***If .... then ....*** (iv) ***if and only if***

CONNECTIVES AND THEIR SYMBOLS

Connective	Symbol	Nature of compound statement
(i) and	$\wedge$	Conjunction
(ii) or	$\vee$	Disjunction
(iii) If ... then ...	$\Rightarrow$	Implication (or conditional)
(iv) if and only if	$\Leftrightarrow$	Biconditional (or equivalence)

EXAMPLES *Each of the following is a compound statement.*

(i) 2 *is a rational number and* $\sqrt{2}$ *is an irrational number.*

(ii) The sum of 3 *and* 5 *is* 7 *or* 8.

(iii) If it rains then the school may be closed.

(iv) A triangle is equiangular if and only if it is equilateral.

The connectives in (i), (ii), (iii) and (iv) are respectively and , or, If ... then, if and only if.

COMPOUNDING OF SENTENCES

1. CONJUNCTION *Two simple sentences connected by the word 'and' are said to form a conjunction.*

We use the symbol '$\wedge$' to denote **'and'**.

TRUTH TABLE FOR $p \wedge q$ *Note that '$p \wedge q$' is true only when each one of the statements p and q is true.*

Based on the above fact, we prepare the truth table for '$p \wedge q$' as under.

Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Remember that:

(i) *The compound statement with 'and' is true if all its component statements are true.*

(ii) *The compound statement with 'and' is false if any of its component statements is false or all of its component statements are false.*

EXAMPLE 1 *Split each of the following compound statements into simple sentences and determine whether it is true or false:*

(i) The grass is green and the sky is blue.

(ii) Agra is in Uttar Pradesh and Shimla is in Punjab.

(iii) All rational numbers are real and all real numbers are not complex numbers.

(iv) $x = 5$ *and* $x = 2$ *are the roots of the equation* $3x^2 - x - 10 = 0$.

SOLUTION (i) Let p: The grass is green.

And q: The sky is blue.

Clearly, each one of p and q is a true statement and therefore, (p and q) is true.

Hence, the given statement is true.

(ii) Let p: Agra is in Uttar Pradesh.
And q: Shimla is in Punjab.
Clearly, p is true and q is false.
Therefore, (p and q) is false.
Hence, the given statement is false.

(iii) Let p: All rational numbers are real.
And, q: All real numbers are not complex numbers.
Clearly, p is true and q is false.
$\therefore$ (p and q) is false.
Hence,the given statement is false.

(iv) Let p: $x = 5$ is a root of the equation $3x^2 - x - 10 = 0$.
And q: $x = 2$ is a root of the equation $3x^2 - x - 10 = 0$.
Clearly, $3 \times 5^2 - 5 - 10 = 60 \neq 0$. So, $x = 5$ is not a root of the equation $3x^2 - x - 10 = 0$.
And, $3 \times 2^2 - 2 - 10 = 0$. So, $x = 2$ is a root of $3x^2 - x - 10 = 0$.
$\therefore$ p is false and q is true.
Consequently, (p and q) is false.
Hence, the given statement is false.

2. DISJUNCTION *Two simple sentences connected by the word 'or' are said to form a disjunction.*

We use the symbol '$\vee$' to denote '**or**'.

Clearly, (p or q) is true when at least one of them is true.

TRUTH TABLE FOR $(p \vee q)$ Clearly, ($p \vee q$) is true only when (p is true or q is true) or both are true.

Truth table for $(p \vee q)$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Remember that:

(i)	*A compound statement with 'or' is true when at least one of them is true.*
(ii)	*A compound statement with 'or' is false when each one of them is false.*

In (p or q) we say that ***exclusive 'or'*** *is used when both p and q are not true simultaneously.*

Otherwise, ***inclusive 'or'*** is used.

EXAMPLE 2 *For each of the following statements, determine whether an* ***inclusive 'or'*** *or* ***exclusive 'or'*** *is used. Give reasons for your answer.*

(i) For identification you need a passport or an Adhar Card.

(ii) The school is closed if it is a holiday or a Sunday.

(iii) $\sqrt{3}$ *is a rational number or an irrational number.*

(iv) Two lines intersect at a point or are parallel.

(v) Students can take Sanskrit or French as their third language.

SOLUTION (i) Here 'or' is inclusive since a person can have both a passport and an Adhar Card for identification.

(ii) Here 'or' is inclusive since the school is closed on holiday as well as on Sunday.

(iii) Let $p : \sqrt{3}$ is a rational number.

And, $q : \sqrt{3}$ is an irrational number.

Clearly, p is false and q is true.

$\therefore$ (p or q) is true and 'or' is *exclusive*.

(iv) Here 'or' is *exclusive* because it is not possible for two lines to intersect and be parallel together.

(v) Here 'or' is *exclusive* because a student cannot take both Sanskrit and French.

OPEN SENTENCES AND THEIR TRUTH SETS

Let $p(x)$ be a declarative sentence involving a variable x and let A be a given set such that for $a \in A$, $p(a)$ is true or false.

Then, $p(x)$ is called an *open sentence* defined on A.

The subset of A consisting of all those values of x which convert $p(x)$ into a true statement is called a *Truth Set* of the *open sentence* $p(x)$.

EXAMPLE 3 *Write down the truth set of each of the following open sentences:*

(i) $p(x): x + 5 < 9, x \in N$.

(ii) $p(x): x + 3 < 3, x \in N$.

(iii) $p(x): x + 5 > 7, x \in R$.

(iv) $p(x): 2x^2 + 5x - 3 = 0, x \in I$.

SOLUTION We have:

(i) Truth Set $= \{x \in N : x + 5 < 9\}$

$= \{x \in N : x < 4\} = \{1, 2, 3\}$.

(ii) Truth Set $= \{x \in N : x + 3 < 3\}$

$= \{x \in N : x < 0\} = \phi$.

(iii) Truth Set $= \{x \in R : x + 5 > 7\}$

$= \{x \in R : x > 2\} =]2, \infty[$.

(iv) $2x^2 + 5x - 3 = 0 \Rightarrow 2x^2 + 6x - x - 3 = 0 \Rightarrow 2x(x + 3) - (x + 3) = 0$

$\Rightarrow (x + 3)(2x - 1) = 0 \Rightarrow x = -3$ or $x = \frac{1}{2}$.

$$\therefore \text{ truth set} = \{x \in I : 2x^2 + 5x - 3 = 0\} = \{-3\} \quad \left[\because \frac{1}{2} \notin I\right].$$

QUANTIFIERS AND QUANTIFIED STATEMENTS

Two Important Symbols:

(i) The symbol '$\forall$' stands *'for all values of'*.
This is known as universal *quantifier*.

(ii) The symbol '$\exists$' stands for *'there exists'*.
The symbol $\exists$ is known as *existential quantifier*.

QUANTIFIED STATEMENT An open sentence with a quantifier becomes a statement, called a quantified statement.

EXAMPLE 4 *Use quantifiers to convert each of the following open sentences defined on N, into a true statement:*

(i) $x + 5 = 8$ *(ii)* $x^2 > 0$ *(iii)* $x + 2 < 4$

SOLUTION (i) $\exists\, x \in N$ such that $x + 5 = 8$ is a true statement, since $x = 3 \in N$ satisfies $x + 5 = 8$.

(ii) $x^2 > 0\, \forall\, x \in N$ is a true statement, since the square of every natural number is positive.

(iii) $\exists\, x \in N$ such that $x + 2 < 4$ is a true statement, since $x = 1 \in N$ satisfies $x + 2 < 4$.

EXAMPLE 5 *Let* $A = \{1, 2, 3, 4\}$. *Examine whether the statements given below are true or false.*

(i) $\exists\, x \in A$ *such that* $x + 3 = 8$.

(ii) $\forall\, x \in A,\ x + 2 < 7$.

(iii) $\exists\, x \leftrightarrow A$ *such that* $x + 1 < 3$.

(iv) $\forall\, x \in A,\ x + 3 \geq 5$.

SOLUTION (i) Clearly, no number in A satisfies $x + 3 = 8$.
$\therefore$ the given statement is false.

(ii) Every number in A satisfies $x + 2 < 7$.
So, the given statement is true.

(iii) Clearly, $x = 1 \leftrightarrow A$ satisfies $x + 1 < 3$.
$\therefore$ the given statement is true.

(iv) Since $x = 1 \in A$ does not satisfy $x + 3 \geq 5$, the given statement is false.

EXERCISE 29B

1. Split each of the following into simple sentences and determine whether it is true or false.

(i) A line is straight and extends indefinitely in both the directions.

(ii) A point occupies a position and its location can be determined.

(iii) The sand heats up quickly in the sun and does not cool down fast at night.

(iv) 32 is divisible by 8 and 12.

(v) $x = 1$ and $x = 2$ are the roots of the equation $x^2 - x - 2 = 0$.

(vi) 3 is rational and $\sqrt{3}$ is irrational.

(vii) All integers are rational numbers and all rational numbers are not real numbers.

(viii) Lucknow is in Uttar Pradesh and Kanpur is in Uttarakhand.

2. Split each of the following into simple sentences and determine whether it is true or false. Also, determine whether an *'inclusive or'* or *'exclusive or'* is used.

(i) The sum of 3 and 7 is 10 or 11.

(ii) $(1 + i)$ is a real or a complex number.

(iii) Every quadratic equation has one or two real roots.

(iv) You are wet when it rains or you are in a river.

(v) 24 is a multiple of 5 or 8.

(vi) Every integer is rational or irrational.

(vii) For getting a driving licence you should have a ration card or a passport.

(viii) 100 is a multiple of 6 or 8.

(ix) Square of an integer is positive or negative.

(x) Sun rises or Moon sets.

3. Find the truth set in case of each of the following open sentences defined on N:

(i) $x + 2 < 10$ (ii) $x + 5 < 4$ (iii) $x + 3 > 2$

4. Let $A = \{2, 3, 5, 7\}$. Examine whether the statements given below are true or false.

(i) $\exists\ x \in A$ such that $x + 3 > 9$.

(ii) $\exists\ x \in A$ such that x is even.

(iii) $\exists\ x \in A$ such that $x + 2 = 6$.

(iv) $\forall\ x \in A$, x is prime.

(v) $\forall\ x \in A$, $x + 2 < 10$

(vi) $\forall\ x \in A$, $x + 4 \geq 11$

ANSWERS (EXERCISE 29B)

1. (i) p: A line is straight.

q: A line extends indefinitely in both the directions.

(p and q) is true.

(ii) p: A point occupies a position.

q: The location of a point can be determined.

(p and q) is true.

(iii) p: The sand heats up quickly in the sun.
q: The hot sand does not cool down fast at night.
(p and q) is true.

(iv) p: 32 is divisible by 8.
q: 32 is divisible by 12.
(p and q) is not true.

(v) p: $x = 1$ is a root of the equation $x^2 - x - 2 = 0$.
q: $x = 2$ is a root of the equation $x^2 - x - 2 = 0$.
(p and q) is not true.

(vi) p: 3 is rational.
q: $\sqrt{3}$ is irrational.
(p and q) is true.

(vii) p: All integers are rational numbers.
q: All rational numbers are not real numbers.
(p and q) is false.

(viii) p: Lucknow is in Uttar Pradesh.
q: Kanpur is in Uttarakhand.
(p and q) is false.

2. (i) p: The sum of 3 and 7 is 10.
q: The sum of 3 and 7 is 11.
(p or q) is true. *'or is exclusive'*

(ii) p: $(1 + i)$ is a real number.
q: $(1 + i)$ is a complex number.
(p or q) is true. *'or is exclusive'*

(iii) p: Every quadratic equation has one real root.
q: Every quadratic equation has two real roots.
(p or q) is false.

(iv) p: You are wet when it rains.
q: You are wet when you are in a river.
(p or q) is true. *'or is inclusive'*

(v) Let p: 24 is a multiple of 5.
and q: 24 is a multiple of 8.
(p or q) is true. *'or is exclusive'*

(vi) Let p: Every integer is rational.
q: Every integer is irrational.
(p or q) is true. *'or is exclusive'*

(vii) Let p: For getting a driving licence you should have a ration card.
q: For getting a driving licence you should have a passport.
(p or q) is true. '*or is inclusive*'

(viii) Let p: 100 is a multiple of 6.
q: 100 is a multiple of 8.
(p or q) is false.

(ix) Let p: Square of an integer is positive.
q: Square of an integer is negative.
(p or q) is false. [**Hint:** 0^2 *is not positive.*]

(x) Let p: Sun rises.
q: Moon sets.
(p or q) is true. '*or is exclusive*'

3. (i) {1, 2, 3, 4, 5, 6, 7} (ii) ϕ (iii) N

4. (i) True (ii) True (iii) False (iv) True (v) True (vi) False

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CONDITIONAL AND BICONDITIONAL STATEMENTS

CONDITIONAL STATEMENTS *Two statements connected by 'if ... then' is called a conditional statement.*

For any statements p and q, the statement '*If p, then q*' is denoted by '$p \Rightarrow q$', which means ***'p implies q'.***

Here p is called the *antecedent* and q is called the *consequent*.

We may write 'if p, then q' in following ways:
(i) p implies q, written as $p \Rightarrow q$.
(ii) p only if q.
(iii) q is a necessary condition for p.
(iv) p is a sufficient condition for q.
(v) $\sim q \Rightarrow \sim p$.

EXAMPLE 1 *Rewrite the following statement, with 'if ..., then' in five different ways conveying the same meaning:*
If a natural number is even, then its square is even.

SOLUTION We may rewrite it in following ways:

(i) A natural number is even implies that its square is even.

(ii) A natural number is even only if its square is even.

(iii) For a natural number to be even it is necessary that its square is even.

(iv) For the square of a natural number to be even, it is sufficient that the number is even.

(v) If the square of a natural number is not even, then the natural number is not even.

EXAMPLE 2 *Write each of the following statements in the form 'if ... then':*

(i) A quadrilateral is a parallelogram if its diagonals bisect each other.

(ii) The banana tree will bloom if it stays warm for a month.

(iii) There is traffic jam whenever it rains.

(iv) It is necessary to have a password to log on to the server.

(v) You can access the website only if you pay a subscription fee.

SOLUTION We can write them as under:

(i) If the diagonals of a quadrialateral bisect each other, then it is a parallelogram.

(ii) If the banana tree stays warm for a month, then it will bloom.

(iii) If it rains, then there is traffic jam.

(iv) If you have a password, then you can log on to the server.

(v) If you can access the website, then you pay a subscription fee.

CONVERSE AND CONTRAPOSITIVE OF STATEMENTS

Let $p \Rightarrow q$. Then,
(i) (*converse* of $p \Rightarrow q$) is $(q \Rightarrow p)$
(ii) (*contrapositive of* $p \Rightarrow q$) *is* $(\sim q \Rightarrow \sim p)$

EXAMPLE 3 *Write the converse and contrapositive of each of the following statements:*

(i) If n is an even number, then n^2 is even.

(ii) If two integers a and b are such that $a > b$, then $(a - b)$ is always a positive integer.

(iii) If a $\triangle ABC$ is right angled at B, then $AB^2 + BC^2 = AC^2$.

(iv) If $\triangle ABC$ and $\triangle DEF$ are congruent, then they are equiangular.

(v) You cannot comprehend geometry if you do not know how to reason deductively.

(vi) Something is cold implies that it has low temperature.

SOLUTION (i) Its **converse** is:

If a number n^2 is even, then n is even.

Its **contrapositive** is:

If a number n^2 is not even, then n is not even.

(ii) Its **converse** is:

If a and b are two integers such that $(a - b)$ is a positive integer, then $a > b$.

Its **contrapositive** is:

If two integers a and b are such that $(a - b)$ is not a positive integer, then a is not greater than b.

(iii) Its **converse** is:

In a $\triangle ABC$, if $AB^2 + BC^2 = AC^2$, then it is right angled at B.

Its **contrapositive** is:

In a $\triangle ABC$, if $(AB^2 + BC^2)$ is not equal to AC^2, then it is not right angled at B.

(iv) Its **converse** is:

If $\triangle ABC$ and $\triangle DEF$ are equiangular, then they are congruent.

Its **contrapositive** is:

If $\triangle ABC$ and $\triangle DEF$ are not equiangular, then they are not congruent.

(v) Its **converse** is:

If you do not know how to reason deductively, then you cannot comprehend geometry.

Its **contrapositive** is:

If you know how to reason deductively, then you can comprehend geometry.

(vi) Given **statement** is:

Something is cold $\Rightarrow$ it has low temperature.

Its **converse** is:

If something has low temperature, then it is cold.

Its **contrapositive** is

If something does not have a low temperature, then it is not cold.

BICONDITIONAL STATEMENT Two simple sentences connected by *'if and only if'* form a *biconditional statement*.

We use the symbol '$\Leftrightarrow$' for *'if and only if'*.

$(p \Leftrightarrow q)$ is the same as $(p \Rightarrow q)$ and $(q \Rightarrow p)$.

EXAMPLE In $\triangle ABC$, $\angle B = \angle C \Leftrightarrow AC = AB$.

This statement is the same as:

'In $\triangle ABC$: $(\angle B = \angle C \Rightarrow AC = AB)$ and $(AC = AB \Rightarrow \angle B = \angle C)$'.

EXAMPLE 4 *Rewrite the following statements in the form:*
'p if and only if q'.
p: If a quadrilateral is equiangular, then it is a rectangle.
q: If a quadrilateral is a rectangle, then it is equiangular.

SOLUTION The required statement is:
A quadrilateral is equiangular if and only if it is a rectangle.

EXAMPLE 5 *Given below are pairs of statements. In each case, combine them using 'if and only if'.*

(i) p: In $\triangle ABC$, $\angle B = \angle C$.
q: In $\triangle ABC$, $AC = AB$.

(ii) p: A and B are two sets such that $A \subseteq B$ *and* $B \subseteq A$.
q: $A = B$.

(iii) p: $\triangle ABC$ *is equilateral.*

q: $\triangle ABC$ *is equiangular.*

(iv) p: $\{a \in R$ *such that* $|a| < 2\}$.

q: $\{a \in R$ *such that* $(a > -2$ *and* $a < 2)\}$.

SOLUTION We can combine the given statements as given below.

(i) In $\triangle ABC$, $\angle B = \angle C \Leftrightarrow AC = AB$.

(ii) For any sets A and B, $A = B \Leftrightarrow (A \subseteq B$ and $B \subseteq A)$.

(iii) A $\triangle ABC$ is equilateral $\Leftrightarrow$ it is equiangular.

(iv) For every real a, $|a| < 2 \Leftrightarrow (a > -2$ and $a < 2)$.

VARIOUS CASES SHOWING VARIOUS CONDITIONS

SOME CONDITIONS ARE NECESSARY BUT NOT SUFFICIENT

EXAMPLES *(i) Having four sides is necessary but not sufficient for being a square (since a rectangle has four sides but it is not a square).*

(ii) Being educated is necessary but not sufficient for getting the job of a teacher (as certain minimum qualifications are required to be recruited as a teacher).

(iii) Having a difference of 2 *between two numbers is necessary but not sufficient for them to be twin-primes (since they must be prime also).*

SOME CONDITIONS ARE SUFFICIENT BUT NOT NECESSARY

EXAMPLES *(i) Having a son is sufficient but not necessary for being a parent (since a parent could have only a daughter).*

(ii) It is obvious that being a multiple of 10 *is sufficient but not necessary for a number to be a composite number.*

(iii) Having 6 *at the unit's place is sufficient but not necessary for a number to be even (since numbers having* 0, 2, 4, 8 *in the unit's place are also even).*

(iv) Having the corresponding sides equal is sufficient but not necessary for two triangles to be similar (since the triangles having the corresponding sides proportional are also similar)

SOME CONDITIONS ARE BOTH SUFFICIENT AND NECESSARY

EXAMPLES *(i) Having four sides is both sufficient and necessary for being a quadrilateral.*

(ii) Having all the three sides of equal measure is both sufficient and necessary for a triangle to be equilateral.

(iii) Having one number as zero out of two numbers to be multiplied is both sufficient and necessary for the product to be zero.

SOME CONDITIONS ARE NEITHER SUFFICIENT NOR NECESSARY

EXAMPLES *(i) Being a tall person is neither sufficient nor necessary for being a successful person.*

(ii) *Being handsome is neither sufficient nor necessary for getting married.*

(iii) *Having both the diagonals equal is neither sufficient nor necessary for a quadrilateral to be a parallelogram.*

EXERCISE 29C

1. Rewrite the following statement in five different ways conveying the same meaning.

   If a given number is a multiple of 6, then it is a multiple of 3.

2. Write each of the following statements in the form 'if .... then':

   (i) A rhombus is a square only if each of its angles measures 90°.

   (ii) When a number is a multiple of 9, it is necessarily a multiple of 3.

   (iii) You get a job implies that your credentials are good.

   (iv) Atmospheric humidity increases only if it rains.

   (v) If a number is not a multiple of 3, then it is not a multiple of 6.

3. Write the converse and contrapositive of each of the following:

   (i) If x is a prime number, then x is odd.

   (ii) If a positive integer n is divisible by 9, then the sum of its digits is divisible by 9.

   (iii) If the two lines are parallel, then they do not intersect in the same plane.

   (iv) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

   (v) If A and B are subsets of X such that $A \subseteq B$, then $(X - B) \subseteq (X - A)$.

   (vi) If $f(2) = 0$, then $f(x)$ is divisible by $(x - 2)$.

   (vii) If you were born in India, then you are a citizen of India.

   (viii) If it rains, then I stay at home.

4. Given below are some pairs of statements. Combine each pair using if and only if:

   (i) p: If a quadrilateral is equiangular, then it is a rectangle.
   q: If a quadrilateral is a rectangle, then it is equiangular.

   (ii) p: If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.
   q: If a number is divisble by 3, then the sum of its digits is divisible by 3.

   (iii) p: A quadrilateral is a parallelogram if its diagonals bisect each other.
   q: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

   (iv) p: If $f(a) = 0$, then $(x - a)$ is a factor of polynomial $f(x)$.
   q: If $(x - a)$ is a factor of polynomial $f(x)$, then $f(a) = 0$.

(v) p: If a square matrix A is invertible, then $|A|$ is nonzero.
q: If A is a square matrix such that $|A|$ is nonzero, then A is invertible.

5. Write each of the following using 'if and only if':

(i) In order to get A grade, it is necessary and sufficient that you do all the homework regularly.

(ii) If you watch television then your mind is free and if your mind is free then you watch television.

ANSWERS (EXERCISE 29C)

1. (i) A given number is a multiple of 6 implies that it is a multiple of 3.
(ii) A given number is a multiple of 6 only if it is a multiple of 3.
(iii) For a given number to be a multiple of 6, it is necessary that it is a multiple of 3.
(iv) For a given number to be a multiple of 3, it is sufficient that the number is a multiple of 6.
(v) If a given number is not a multiple of 3, then it is not a multiple of 6.

2. (i) If each angle of a rhombus measures 90°, then it is a square.
(ii) If a number is a multiple of 9, then it is a multiple of 3.
(iii) If you get a job, then your credentials are good.
(iv) If it rains, then atmospheric humidity increases.
(v) If a number is a multiple of 6, then it is a multiple of 3.

3. (i) Converse If x is an odd number, then it is prime.
Contrapositive If x is not an odd number, then it is not prime.

(ii) Converse If the sum of the digits of a positive integer n is divisible by 9, then it is divisible by 9.
Contrapositive If the sum of the digits of a positive integer n is not divisble by 9, then it is not divisible by 9.

(iii) Converse If the two lines do not intersect in the same plane, then they are parallel.
Contrapositive If the two lines intersect in the same plane, then they are not parallel.

(iv) Converse If a quadrilateral is a parallelogram, then its diagonals bisect each other.
Contrapositive If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.

(v) Converse If A and B are subsets of X such that $(X-B) \subseteq (X-A)$, then $A \subseteq B$.
Contrapositive If A and B are subsets of X such that $(X-B)$ is not a subset of $(X-A)$, then A is not a subset of B.

(vi) Converse If $f(x)$ is divisible by $(x-2)$, then $f(2) = 0$.
Contrapositive If $f(x)$ is not divisible by $(x-2)$, then $f(2)$ is not equal to 0.

(vii) Converse If you are a citizen of India, then you were born in India.

Contrapositive If you are not a citizen of India, then you were not born in India.

(viii) Converse If I stay at home, then it rains.

Contrapositive If I don't stay at home, then it does not rain.

4. (i) A quadrilateral is a rectangle if and only if it is equiangular.

(ii) A number is divisible by 3 if and only if the sum of its digits is divisible by 3.

(iii) A quadrilateral is a parallelogram if and only if its diagonals bisect each other.

(iv) $(x - a)$ is a factor of polynomial $f(x)$ if and only if $f(a) = 0$.

(v) A square matrix A is invertible if and only if $|A|$ is nonzero.

5. (i) You get an A grade if and only if you do your homework regularly.

(ii) You watch television if and only if your mind is free.

VALIDATING STATEMENTS

We shall list some general rules for checking whether a given compound statement is true or not.

Rule 1. *Statements with 'And':*

Let p and q be two statements. Then, in order to prove that *'p and q'* is true, the following steps are taken.

Step 1. Show that p is true.

Step 2. Show that q is true.

Rule 2. *Statements with 'or':*

Let p and q be two statements. Then, in order to prove that *'p or q'* is true, we proceed as under.

Case 1. Assume that p is false and show that q is true.

Case 2. Assume that q is false and show that p is true.

Rule 3. *Statements with 'If .... then':*

Let p and q be two statements. Then, in order to prove that *'$p \Rightarrow q$'* is true, we need to prove any one of the following.

DIRECT METHOD *Assume that p is true and prove that q is true.*

CONTRAPOSITIVE METHOD *Assume that q is false and prove that p is false.*

Rule 4. *Statements with 'If and only if':*

In order to prove that *'$p \Rightarrow q$', we need to show that:*
(i) *If p is true, then q is true.*
(ii) *If q is true, then p is true.*

SOLVED EXAMPLES

EXAMPLE 1 *Prove that the following statement is true:*
If $x, y \in Z$ such that x and y are odd, then xy is odd.

SOLUTION Let p: $x, y \in Z$ such that x and y are odd.
And q: xy is odd.
Then, we have to prove that xy is odd.

DIRECT METHOD

We assume that p is true and show that q is true.
p is true means x and y are odd integers. Then,

$$x = (2m+1) \text{ for some integer } m$$

and $y = (2n+1)$ for some integer n.

$$\therefore \quad xy = (2m+1)(2n+1)$$
$$= (4mn + 2m + 2n + 1)$$
$$= 2(2mn + m + n) + 1, \text{ which is clearly odd.}$$

Thus, $p \Rightarrow q$.
Hence, the given statement is true.

CONTRAPOSITIVE METHOD

We will show that $\sim q \Rightarrow \sim p$.
Here $\sim q$: It is false that both x and y are odd.
This means at least one of x and y is even.
Let x be even. Then, $x = 2n$ for some integer n.
$\therefore \quad xy = 2ny$ for some integer n.
This shows that xy is even.
Thus, $\sim p$ is true $[\because \sim p : xy$ is even$]$.
$\therefore \quad \sim q \Rightarrow \sim p$.
So, the given statement is true.

EXAMPLE 2 *Prove that the necessary and sufficient condition for an integer n to be odd is that n^2 is odd.*

SOLUTION Let p: The integer n is odd.
And q: n^2 is odd.

First we prove that $p \Rightarrow q$.

Let n be odd. Then, $n = (2k+1)$ for some integer k.

Then, $n^2 (2k+1)^2 = (4k^2 + 4k + 1) = 2(2k^2 + 2k) + 1$

Thus, n^2 is 1 more than an even number and therefore, it is odd.

Thus, $p \Rightarrow q$.

Now, in order to prove that $q \Rightarrow p$, it is sufficient to show that $\sim p \Rightarrow \sim q$ [*Contrapositive method*].

Clearly, $\sim p$: The integer n is even.

Let $n = 2k$ for some integer k.

Then, $n^2 = 4k^2$, which is even.

$\therefore$ n is even $\Rightarrow n^2$ is even.

Consequently, $\sim p \Rightarrow \sim q$ and therefore, $q \Rightarrow p$.

Hence, $p \Leftrightarrow q$, i.e., the integer n is odd if and only if n^2 is odd.

PROVING RESULTS BY CONTRADICTION

In order to check whether a given statement p is true, we assume that p is false, i.e., ~ p is true.

From this we arrive at a result which contradicts our assumption. Hence, we conclude that p is true.

EXAMPLE 3 *By using the method of contradiction verify that p: $\sqrt{5}$ is irrational.*

PROOF If possible, let $\sqrt{5}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{5} = \frac{a}{b} \Rightarrow 5 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 5b^2 = a^2$ ... (i)

$\Rightarrow 5$ divides a^2 [$\because$ 5 divides $5b^2$]

$\Rightarrow 5$ divides a [$\because$ 5 is prime and divides $a^2 \Rightarrow 5$ divides a].

Let $a = 5c$ for some integer C.

Putting $a = 5c$ in (i), we get:

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

$\Rightarrow 5$ divides b^2 [$\because$ 5 *divides* $5c^2$]

$\Rightarrow 5$ divides b [$\because$ 5 is prime and divides $b^2 \Rightarrow 5$ divides b].

Thus, 5 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

Hence, $\sqrt{5}$ is irrational.

EXAMPLE 4 *By using the method of contradiction, prove that the sum of an irrational number and a rational number is irrational.*

PROOF Let $\sqrt{a}$ be irrational and b be rational.

Then, we have to prove that $(\sqrt{a}+b)$ is irrational.

If possible, let $(\sqrt{a}+b)$ be rational. Then,

$(\sqrt{a}+b)$ is rational, b is rational.

$\Rightarrow \{(\sqrt{a}+b)-b\}$ is rational [$\because$ difference of rationals is rational]

$\Rightarrow \sqrt{a}$ is rational.

This contradicts the fact that $\sqrt{a}$ is irrational.

Since the contradiction arises by assuming that $(\sqrt{a}+b)$ is rational, hence $(\sqrt{a}+b)$ is irrational.

EXAMPLE 5 *Using contradiction method, check the validity of the following statement:*
If n is a real number with $n > 3$, then $n^2 > 9$.

PROOF Let n be a real number such that $n > 3$ and if possible, let n^2 be not greater than 9.

Then, $n^2 \leq 9$.

Let $n^2 = (9-a)$, where a is a real number such that $a \geq 0$.

Then, $n = \sqrt{n^2} = \sqrt{9-a} \leq 3 \quad [\because \quad (9-a) \leq 9]$.

But, this is a contradiction since $n > 3$.

Since the contradiction arises by assuming that n^2 is not greater than 9, therefore, $n^2 > 9$.

Hence, the given statement is valid.

EXAMPLE 6 *Consider the statement:*
p: If x a real number such that $x^3 + 4x = 0$, then $x = 0$.
Prove that p is a true statement, using:
(i) direct method
(ii) method of contradiction
(iii) method of contrapositive

SOLUTION (i) Direct Method:

Let $x^3 + 4x = 0$, where $x \in R$. Then,

$x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0$

$\Rightarrow x = 0 \quad [\because \quad x^2 + 4 \neq 0 \text{ for } x \in R]$

Hence, p is a true statement.

(ii) Method of contradiction:

If possible, let $x^3 + 4x = 0$ and $x \neq 0$. Then,

$x(x^2 + 4) = 0$ and $x \neq 0 \Rightarrow x^2 + 4 = 0$.

But, this is a contradiction, since $x^2 + 4 \neq 0$ for $x \in R$.

Since, the contradiction arises by assuming that

$x^2 + 4 = 0$ and $x \neq 0$, so $x = 0$.

Hence, $x^3 + 4x = 0 \Rightarrow 0$ is a true statement.

(iii) Method of contrapositive:

We have to prove that $x^3 + 4x = 0 \Rightarrow x = 0$.

Let p: $x^3 + 4x = 0$ and q: $x = 0$.

We shall prove that $\sim q \Rightarrow \sim p$.
Let $x \neq 0$. Then, $x^3 + 4x = x(x^2 + 4) \neq 0$ [$\because$ $x \neq 0$ and $x^2 + 4 \neq 0$].
Thus, $\sim q \Rightarrow \sim p$ and therefore, $p \Rightarrow q$.

EXERCISE 29D

1. Let p: *If x is an integer and x^2 is even, then x is even.*
Using the method of contrapositive, prove that p is true.

2. Consider the statement:
q: *For any real numbers a and b, $a^2 = b^2 \Rightarrow a = b$.*
By giving a counter-example, prove that q is false.

3. By giving a counter-example, show that the statement is false:
p: If n is an odd positive integer, then n is prime.

4. Use contradiction method to prove that:
p: *$\sqrt{3}$ is irrational*
is a true statement.

5. By giving a counter-example, show that the following statement is false:
p: *If all the sides of a triangle are equal, then the triangle is obtuse angled.*

❐

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Statistics

INTRODUCTION

In earlier classes we have studied the methods of representing data graphically and in tabular form.

Mean, median and mode are three measures of central tendency, giving us a rough idea about the points where data are centred.

In order to have a better idea as to how the data are scattered, we make a study of (i) mean deviation and (ii) standard deviation.

MEAN DEVIATIONS (MD)

MEAN DEVIATION ABOUT THE MEAN *The AM of the numerical deviations of the observations from the mean of the data is called the mean deviation about the mean.*

MEAN DEVIATION ABOUT THE MEDIAN *The AM of the numerical deviations of the observations from the median of the data is called the mean deviation about the median.*

MEAN DEVIATIONS FOR UNGROUPED DATA

Let $x_1, x_2, x_3, \ldots, x_n$ be the given n observations. Let $\overline{x}$ be the AM and M be the median. Then,

$$\text{(i)}\ \ \text{MD}(\overline{x}) = \frac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}, \text{ where } \overline{x} = \text{mean}.$$

$$\text{(ii)}\ \ \text{MD}(M) = \frac{\sum_{i=1}^{n} |x_i - M|}{n}, \text{ where } M = \text{median}.$$

SUMMARY

$$\text{(i)}\ \ \text{MD}(\overline{x}) = \frac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}, \text{ where } \overline{x} = \text{mean}.$$

$$\text{(ii)}\ \ \text{MD}(M) = \frac{\sum_{i=1}^{n} |x_i - M|}{n}, \text{ where } M = \text{median}.$$

EXAMPLE 1 *Find the mean deviation about the mean for the following data:*

$15, 17, 10, 13, 7, 18, 9, 6, 14, 11$

SOLUTION Let the mean of the given data be $\bar{x}$. Then,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{120}{10} = 12 \quad [\because \ n = 10].$$

The values of $(x_i - \bar{x})$ are:

$3, 5, -2, 1, -5, 6, -3, -6, 2, -1.$

So, the values of $|x_i - \bar{x}|$ are:

$3, 5, 2, 1, 5, 6, 3, 6, 2, 1.$

$$\therefore \quad \text{MD}(\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} = \frac{34}{10} = 3.4.$$

Hence, $\text{MD}(\bar{x}) = 3.4$.

EXAMPLE 2 *Find the mean deviation about the median for the data given below:*

$11, 3, 8, 7, 5, 14, 10, 2, 9$

SOLUTION Arranging the given data in an ascending order, we get:

$2, 3, 5, 7, 8, 9, 10, 11, 14.$

Here $n = 9$, which is odd.

$$\therefore \quad \text{median} = \frac{1}{2}(n+1)\text{th observation}$$

$$= \frac{1}{2}(9+1)\text{th observation} = 5\text{th observation} = 8.$$

Thus, $M = 8$.

The values of $(x_i - M)$ are:

$-6, -5, -3, -1, 0, 1, 2, 3, 6.$

$$\therefore \quad \sum_{i=1}^{9} |x_i - M| = (6+5+3+1+0+1+2+3+6) = 27$$

$$\Rightarrow \quad \text{MD}(M) = \frac{\sum_{i=1}^{9} |x_i - M|}{9} = \frac{27}{9} = 3.$$

Hence, $\text{MD}(M) = 3$.

EXAMPLE 3 *Find the mean deviation about the median for the data given below.*

$45, 36, 50, 60, 53, 46, 51, 48, 72, 42$

SOLUTION Arranging the given data in an ascending order, we get:

$36, 42, 45, 46, 48, 50, 51, 53, 60, 72.$

Here $n = 10$, which is even.

$$\therefore \quad \text{median} = \frac{1}{2}\cdot\left\{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}\right\}$$

$$= \frac{1}{2}(5\text{th observation} + 6\text{th observation})$$

$$= \frac{1}{2}(48+50) = \frac{98}{2} = 49.$$

Thus, $M = 49$.

The values of $(x_i - M)$ are

$$-13, -7, -4, -3, -1, 1, 2, 4, 11, 23.$$

$$\therefore \quad \sum_{i=1}^{10} |x_i - M| = (13+7+4+3+1+1+2+4+11+23) = 69$$

$$\Rightarrow \quad \text{MD}(M) = \frac{\sum_{i=1}^{10} |x_i - M|}{10} = \frac{69}{10} = 6.9.$$

MEAN DEVIATION FOR DISCRETE FREQUENCY DISTRIBUTION

Let the given data consist of n distinct values $x_1, x_2, \ldots, x_n$ occurring with frequencies $f_1, f_2, \ldots, f_n$ respectively.

(i) Mean deviation about mean:

We have:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \Rightarrow \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \text{ where } N = \sum_{i=1}^{n} f_i .$$

$$\therefore \quad \text{MD}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}.$$

(ii) Mean deviation about median:

Let $N = \sum_{i=1}^{n} f_i$. First we find the cumulative frequencies and then, identify the observation whose cumulative frequency is equal to or just greater than $\frac{N}{2}$.

This value gives median. Then,

$$\text{MD}(M) = \frac{\sum_{i=1}^{n} f_i |x_i - M|}{N}.$$

SUMMARY

For discrete frequency distribution, we have

(i) $\text{MD}(\bar{x}) = \dfrac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$, where $\sum_{i=1}^{n} f_i = N$.

(ii) $\text{MD}(M) = \dfrac{\sum_{i=1}^{n} f_i |x_i - M|}{N}$, where $\sum_{i=1}^{n} f_i = N$.

EXAMPLE 4 *Find the mean deviation about the mean for the following data:*

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

SOLUTION We have

$$N = \sum_{i=1}^{6} f_i = (6 + 8 + 15 + 25 + 8 + 4) = 66.$$

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{N} = \frac{(6 \times 3) + (8 \times 5) + (15 \times 7) + (25 \times 9) + (8 \times 11) + (4 \times 13)}{66}$$

$$= \frac{528}{66} = 8.$$

Now, we prepare the table given below:

x_i	f_i	$\lvert x_i - \bar{x} \rvert$	$\sum_{i=1}^{6} f_i \lvert x_i - \bar{x} \rvert$
3	6	5	30
5	8	3	24
7	15	1	15
9	25	1	25
11	8	3	24
13	4	5	20
	$N = 66$		sum = 138

$$\therefore \quad \text{MD}(\bar{x}) = \frac{\sum_{i=1}^{6} f_i |x_i - \bar{x}|}{N} = \frac{138}{66} = \frac{23}{11} = 2.09.$$

Hence, mean deviation about the mean = 2.09.

EXAMPLE 5 *Find the mean deviation about the median for the following data:*

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4

SOLUTION We have

$$N = \sum_{i=1}^{6} f_i = (6 + 8 + 15 + 3 + 8 + 4) = 44.$$

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{N} = \frac{(6 \times 3) + (8 \times 5) + (15 \times 7) + (3 \times 9) + (8 \times 11) + (4 \times 13)}{44}$$

$$= \frac{(18 + 40 + 105 + 27 + 88 + 52)}{44} = \frac{330}{44} = \frac{15}{2} = 7.5.$$

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4
cf	6	14	29	32	40	44

$\therefore$ $N = 44$, which is even.

$\therefore$ $\text{median} = \frac{1}{2} \cdot \left\{ \frac{N}{2}\text{th observation} + \left(\frac{N}{2} + 1 \right)\text{th observation} \right\}$

$$= \frac{1}{2}\{22\text{nd observation} + 23\text{rd observation}\}$$

$$= \frac{1}{2}(7 + 7) = 7.$$

Thus, $M = 7$.

Now, we have:

$\lvert x_i - M \rvert$	4	2	0	2	4	6
f_i	6	8	15	3	8	4
$f_i \lvert x_i - M \rvert$	24	16	0	6	32	24

$\therefore$ $\sum_{i=1}^{6} f_i = 44$ and $\sum_{i=1}^{6} f_i |x_i - M| = 102$.

$\therefore$ $\text{MD}(M) = \frac{\sum_{i=1}^{6} f_i \, |x_i - M|}{N} = \frac{102}{44} = 2.32.$

Hence, the mean deviation about the median is 2.32.

MEAN DEVIATION ABOUT THE MEAN AND MEDIAN FOR CONTINUOUS FREQUENCY DISTRIBUTION

I. MEAN DEVIATION ABOUT THE MEAN FOR GROUPED DATA

Short Cut Method (Step-deviation method):

Step 1 *Find the class mark x_i of each class.*

Step 2 *Assume that the mean is A.*

Step 3 *Find deviations, $d_i = \frac{(x_i - A)}{h}$, where h = class size.*

Step 4 *Calculate,* $\bar{x} = A + \left\{\frac{f_i\, d_i}{N} \times h\right\}$, *where* $N = \sum f_i$.

Step 5 *Calculate,* $MD\,(\bar{x}) = \frac{\sum f_i\, |x_i - \bar{x}\,|}{N}$.

EXAMPLE 6 *Find the mean deviation about the mean for the following data:*

Marks obtained	10–20	20–30	30–40	40–50	50–60	60–70
Number of students	8	6	12	5	2	7

SOLUTION Here $h = 10$. Let the assumed mean be $A = 35$.
Then, we prepare the table given below.

Marks obtained	Number of students (f_i)	Midpoint x_i	$d_i = \frac{x_i - 35}{10}$	$f_i\, d_i$	$\lvert x_i - \bar{x}\,\rvert$	$f_i \times \lvert x_i - \bar{x}\,\rvert$
10–20	8	15	–2	–16	22	176
20–30	6	25	–1	–6	12	72
30–40	12	$35 = A$	0	0	2	24
40–50	5	45	1	5	8	40
50–60	2	55	2	4	18	36
60–70	7	65	3	21	28	196
	$N = \sum f_i = 40$			$\sum f_i\, d_i = 8$		$\sum f_i \times \lvert x_i - \bar{x}\,\rvert = 544$

$$N = \sum f_i = 40,\ \bar{x} = A + \left\{\frac{\sum f_i\, d_i}{N} \times h\right\} = 35 + \left\{\frac{8}{40} \times 10\right\} = 37.$$

$$\therefore \quad \text{MD}\,(\bar{x}) = \frac{\sum f_i \times |x_i - \bar{x}\,|}{N} = \frac{544}{40} = \frac{136}{10} = 13.6.$$

II. MEAN DEVIATION ABOUT THE MEDIAN FOR GROUPED DATA

Short Cut Method:

Step 1 Find the class mark x_i of each class.

Step 2 Find $N = \sum f_i$.

Step 3 Now, median class is the class interval whose cumulative frequency is greater than or equal to $(N/2)$.

Step 4 Calculate median by using the formula

$$M = median = \left\{ L + \frac{\left(\frac{N}{2} - c\right)}{f} \times h \right\}, \text{ where}$$

L = lower limit of the median class;
f = frequency of the median class;
h = width of the median class;
and c = cumulative frequency of the class just preceding the median class.

Step 5 Calculate MD $(M) = \dfrac{\sum f_i \, |x_i - M|}{N}$.

EXAMPLE 7 *Calculate the mean deviation about the median for the following data:*

Height (*in* cm)	95–105	105–115	115–125	125–135	135–145	145–155
Number of boys	9	13	25	30	13	10

SOLUTION First we find the median.

Class	Frequency f_i	Cumulative frequency (cf)	Midpoint x_i
95–105	9	9	100
105–115	13	22	110
115–125	25	47	120
125–135	30	77	130
135–145	13	90	140
145–155	10	100	150
	$N = \sum f_i = 100$		

Thus $N = 100$ and therefore, $\dfrac{N}{2} = 50$

$\Rightarrow$ median class is 125–135

$\Rightarrow$ $L = 125$, $f = 30$, $h = 10$ and $c = 47$.

$\therefore$ $\text{median} = L + \dfrac{\left(\frac{N}{2} - c\right)}{f} \times h$

$$= \left\{125 + \frac{(50 - 47)}{30} \times 10\right\} = (125 + 1) = 126.$$

$\therefore$ $M = 126$.

Now, we prepare the table given below.

f_i	x_i	$\lvert x_i - M\rvert$	$f_i \times \lvert x_i - M\rvert$
9	100	26	234
13	110	16	208
25	120	6	150
30	130	4	120
13	140	14	182
10	150	24	240
$N = 100$			1134

$$\therefore \quad \text{MD}(M) = \frac{\sum f_i \times |x_i - M|}{N} = \frac{1134}{100} = 11.34.$$

Hence, the mean deviation about the median is 11.34.

EXAMPLE 8 *Calculate the mean deviation about the median for the following data:*

Class	16–20	21–25	26–30	31–35	36–40	41–45	46–50	51–55
Frequency	5	6	12	14	26	12	16	9

SOLUTION Converting the given series into an exclusive series, we prepare the table, given below:

Class	Frequency (f_i)	*cf*	Midpoint (x_i)
15.5–20.5	5	5	18
20.5–25.5	6	11	23
25.5–30.5	12	23	28
30.5–35.5	14	37	33
35.5–40.5	26	63	38
40.5–45.5	12	75	43
45.5–50.5	16	91	48
50.5–55.5	9	100	53
	$N = 100$		

Thus $N = 100$ and therefore, $\frac{N}{2} = 50$

$\Rightarrow$ median class is 35.5–40.5

$\Rightarrow$ $L = 35.5$, $f = 26$, $h = 5$ and $c = 37$.

$$\therefore \quad \text{median} = L + \frac{\left(\frac{N}{2} - c\right)}{f} \times h$$

$$= \left\{ 35.5 + \frac{(50 - 37)}{26} \times 5 \right\} = (35.5 + 2.5) = 38.$$

Thus, $M = 38$.

Now, we prepare the table given below.

f_i	x_i	$\lvert x_i - M \rvert$	$f_i \times \lvert x_i - M \rvert$
5	18	20	100
6	23	15	90
12	28	10	120
14	33	5	70
26	38	0	0
12	43	5	60
16	48	10	160
9	53	15	135
$N = 100$			735

Thus, $\sum f_i \times \lvert x_i - M \rvert = 735$ and $N = 100$.

$\therefore \quad \text{MD}(M) = \dfrac{\sum f_i \times \lvert x_i - M \rvert}{N} = \dfrac{735}{100} = 7.35.$

Hence, the mean deviation about the median is 7.35.

EXERCISE 30A

Find the mean deviation about the mean for the following data:

1. 7, 8, 4, 13, 9, 5, 16, 18

2. 39, 72, 48, 41, 43, 55, 60, 45, 54, 43

3. 17, 20, 12, 13, 15, 16, 12, 18, 15, 19, 12, 11

Find the mean deviation about the median for the following data:

4. 12, 5, 14, 6, 11, 13, 17, 8, 10

5. 4, 15, 9, 7, 19, 13, 6, 21, 8, 25, 11

6. 34, 23, 46, 37, 40, 28, 32, 50, 35, 44

7. 70, 34, 42, 78, 65, 45, 54, 48, 67, 50, 56, 63

Find the mean deviation about the mean for the following data:

8.

x_i	6	12	18	24	30	36
f_i	5	4	11	6	4	6

9.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

10.

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

Find the mean deviation about the median for the following data:

11.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

12.

x_i	5	7	9	11	13	15	17
f_i	2	4	6	8	10	12	8

13.

x_i	10	15	20	25	30	35	40	45
f_i	7	3	8	5	6	8	4	9

Find the mean deviation about the mean for the following data:

14.

Mark	0–10	10–20	20–30	30–40	40–50	50–60
Number of students	6	8	14	16	4	2

15.

Height (in cm)	95–105	105–115	115–125	125–135	135–145	145–155
Number of boys	9	16	23	30	12	10

16.

Class	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	3	7	12	15	8	3	2

Find the mean deviation about the median for the following data:

17.

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	6	7	15	16	4	2

18.

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	6	8	11	18	5	2

1. 4.25	**2.** 8.2	**3.** 2.5	**4.** 3
5. 5.36	**6.** 6.5	**7.** 10.5	**8.** 8
9. 2.3	**10.** 2.09	**11.** 5.1	**12.** 2.72
13. 10.1	**14.** 10.24	**15.** 11.6	**16.** 11.36
17. 10.16	**18.** 10.8		

VARIANCE AND STANDARD DEVIATION

VARIANCE *Mean of the squares of the deviations from the mean is called the variance, to be denoted by* σ^2.

The variance of n observations $x_1, x_2, \ldots, x_n$ is given by $\sigma^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$.

STANDARD DEVIATION *The positive square root of variance is called standard deviation and it is denoted by* σ.

$$\sigma = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}.$$

EXAMPLE 1 *Find the mean, variance and the standard deviation for the following data:*
5, 9, 8, 12, 6, 10, 6, 8

SOLUTION Here $n = 8$.

Mean, $\bar{x} = \frac{1}{8}(5 + 9 + 8 + 12 + 6 + 10 + 6 + 8) = \frac{64}{8} = 8.$

The values of $(x_i - \bar{x})$ are

$-3, 1, 0, 4, -2, 2, -2, 0.$

$\therefore \quad (x_i - \bar{x})^2 = (9 + 1 + 0 + 16 + 4 + 4 + 4 + 0) = 38.$

$\therefore \quad$ variance, $\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{38}{8} = \frac{19}{4} = 4.75.$

and, standard deviation, $\sigma = \sqrt{4.75} = 2.17.$

$\therefore \quad$ mean = 8, variance = 4.75 and standard deviation = 2.17.

Short Cut Method

For simple ungrouped data, we have

$$\sigma^2 = \frac{1}{n} \cdot \left\{ \sum_{i=1}^{n} (x_i - \bar{x})^2 \right\}$$

$$= \frac{1}{n} \cdot \left\{ \sum_{i=1}^{n} (x_i^2 - 2x_i \bar{x} + \bar{x}^2) \right\}$$

$$= \frac{\Sigma x_i^2}{n} - 2\bar{x}\left(\frac{\Sigma x_i}{n}\right) + \frac{n\bar{x}^2}{n}$$

$$= \frac{\sum x_i^2}{n} - 2\,\bar{x}\cdot\bar{x} + \bar{x}^2 = \frac{\sum x_i^2}{n} - \bar{x}^2.$$

Thus, for short cut method, we have

$$\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2.$$

EXAMPLE 2 *Find the mean, standard deviation and variance of first n natural numbers.*

SOLUTION First n natural numbers are 1, 2, 3, …, n.

$$\text{Mean, } \bar{x} = \frac{(1 + 2 + 3 + \ldots + n)}{n} = \frac{1}{n}\cdot\frac{1}{2}n(n+1) = \frac{1}{2}(n+1)$$

$$\left[\because\ (1 + 2 + 3 + \ldots + n) = \frac{1}{2}n(n+1)\right]$$

$$\therefore \quad \text{variance, } \sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$= \frac{\sum n^2}{n} - \left\{\frac{1}{2}(n+1)\right\}^2$$

$$= \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{4}(n+1)^2$$

$$\left[\because\ \sum n^2 = \frac{1}{6}n(n+1)(2n+1)\right]$$

$$= \left\{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}\right\}$$

$$= (n+1)\cdot\left\{\frac{(2n+1)}{6} - \frac{(n+1)}{4}\right\}$$

$$= \frac{(n+1)(n-1)}{12} = \frac{(n^2-1)}{12}.$$

$$\therefore \quad \text{variance, } \sigma^2 = \frac{(n^2-1)}{12}.$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{n^2-1}{12}} = \frac{1}{2}\cdot\sqrt{\frac{n^2-1}{3}}.$$

EXAMPLE 3 *Find the mean, standard deviation and variance of first* 10 *multiples of* 3.

SOLUTION First 10 multiples of 3 are 3, 6, 9, 12, …, 30.

This is an AP with $a = 3$, $n = 10$ and $l = 30$.

$$\therefore \quad \text{sum} = \frac{n}{2}(a + l) = \frac{10}{2}\cdot(3 + 30) = 165.$$

$$\therefore \quad \text{mean, } \bar{x} = \frac{165}{10} = 16.5.$$

$$\text{Variance, } \sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$= \left\{ \frac{3^2 + 6^2 + 9^2 + \ldots + 30^2}{10} - (16.5)^2 \right\}$$

$$= \frac{3^2 \times \{1^2 + 2^2 + 3^2 + \ldots + 10^2\}}{10} - (16.5)^2$$

$$= \frac{9 \times 10 \times (10+1)(2 \times 10 + 1)}{6 \times 10} - (16.5)^2$$

$$\left[\because \ \Sigma n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \left(\frac{9 \times 10 \times 11 \times 21}{6 \times 10} - 272.25 \right) = \left(\frac{693}{2} - 272.25 \right)$$

$$= (346.5 - 272.25) = 74.25.$$

$\therefore$ variance, $\sigma^2 = 74.25$.

Standard deviation, $\sigma = \sqrt{74.25} = 8.61$.

VARIANCE AND STANDARD DEVIATION OF A DISCRETE FREQUENCY DISTRIBUTION

Let the given frequency distribution be:

Variable	x_1	x_2	x_3	...	x_n
Frequency	f_1	f_2	f_3	...	f_n

Then, we define:

(i) *Variance,* $\sigma^2 = \frac{1}{N} \cdot \sum_{i=1}^{n} f_i (x_i - \bar{x})^2$, where $N = \sum_{i=1}^{n} f_i$

(ii) *Standard deviation,* $\sigma = \sqrt{\frac{1}{N} \cdot \Sigma f_i (x_i - \bar{x})^2}$

SUMMARY

$$\text{Variance, } \sigma^2 = \frac{1}{N} \cdot \sum_{i=1}^{n} f_i (x_i - \bar{x})^2$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

EXAMPLE 4 *Find the variance and standard deviation for the following data:*

x_i	10	15	18	20	25
f_i	3	2	5	8	2

SOLUTION We may prepare the table given below:

x_i	f_i	$f_i\ x_i$	$(x_i - \bar{x}) = (x_i - 18)$	$(x_i - \bar{x})^2$	$f_i\ (x_i - \bar{x})^2$
10	3	30	–8	64	192
15	2	30	–3	9	18
18	5	90	0	0	0
20	8	160	2	4	32
25	2	50	7	49	98
$\bar{x} = \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{20} = 18$					$\Sigma f_i\ (x_i - \bar{x})^2 = 340$

$$\therefore \quad N = \sum_{i=1}^{5} f_i = 20, \quad \sum_{i=1}^{5} f_i\ (x_i - \bar{x})^2 = 340.$$

$$\therefore \quad \text{variance, } \sigma^2 = \frac{1}{N} \cdot \Sigma f_i\ (x_i - \bar{x})^2 = \left(\frac{1}{20} \times 340\right) = 17.$$

Standard deviation, $\sigma = \sqrt{17} = 4.12$.

Short Cut Method:

Let A be the assumed mean and let $d_i = (x_i - A)$.

Then, $\sigma^2 = \left\{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2\right\}$.

EXAMPLE 5 *Find the mean, variance and standard deviation for the following data using short cut method:*

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

SOLUTION Let the assumed mean be $A = 64$.

Then, we may present the table given below:

x_i	f_i	$d_i = (x_i - A) = (x_i - 64)$	d_i^2	$f_i\ d_i$	$f_i\ d_i^2$
60	2	–4	16	–8	32
61	1	–3	9	–3	9
62	12	–2	4	–24	48
63	29	–1	1	–29	29
$64 = A$	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100			0	286

$\therefore \quad N = \Sigma f_i = 100, \ \Sigma f_i d_i = 0$ and $\Sigma f_i d_i^2$

$\therefore \quad \text{mean} = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i} = \left(64 + \dfrac{0}{100}\right) = 64.$

$$\text{Variance, } \sigma^2 = \frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2$$

$$= \frac{286}{100} - \left(\frac{0}{100}\right)^2 = \frac{286}{100} = 2.86.$$

$\therefore \quad$ standard deviation, $\sigma = \sqrt{2.86} = 1.69.$

When Mean is a Decimal Fraction

In this case, we use the formulae given below.

(i) Variance, $\sigma^2 = \dfrac{1}{N^2} \cdot \left\{ N \cdot \sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i \right)^2 \right\}$

(ii) Standard deviation, $\sigma = \dfrac{1}{N} \cdot \sqrt{N \cdot \sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i \right)^2}$

EXAMPLE 6 *Find the standard deviation for the following data:*

x_i	3	8	13	18	23
f_i	6	10	14	10	10

SOLUTION We have

$$\text{Mean, } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{(18 + 80 + 182 + 180 + 230)}{50} = \frac{690}{50} = 13.8.$$

This value being in decimal form, the calculation will become tedious.

So, we use the formula, $\sigma = \dfrac{1}{N} \cdot \sqrt{N \cdot \Sigma f_i x_i^2 - (\Sigma f_i)^2}$.

Now, we prepare the table given below:

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
3	6	18	9	54
8	10	80	64	640
13	14	182	169	2366
18	10	180	324	3240
23	10	230	529	5290
	50	690		11590

$\therefore \quad N = \Sigma f_i = 50, \quad \Sigma f_i x_i = 690 \quad \text{and} \quad \Sigma f_i x_i^2 = 11590$

$\therefore \quad$ standard deviation, $\sigma = \frac{1}{N} \cdot \{\sqrt{N \cdot \Sigma f_i x_i^2 - (\Sigma f_i x_i)^2}\}$

$\Rightarrow \quad \sigma = \frac{1}{50} \cdot \sqrt{50 \times 11590 - (690)^2} = \frac{1}{50} \times \sqrt{579500 - (690)^2}$

$= \frac{1}{50} \sqrt{579500 - 476100} = \frac{1}{50} \times \sqrt{103400} = \frac{321.5}{50} = 6.43.$

Hence, $\sigma = 6.43$.

Shorter Method For Standard Deviation of Continuous Frequency Distribution

Let A be the assumed mean and let $y_i = \frac{(x_i - A)}{h}$.

Then, we have:

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \cdot \left\{ N \cdot \sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i \right)^2 \right\}$$

$$\text{Standard deviation, } \sigma = \frac{h}{N} \cdot \sqrt{N \cdot \sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i \right)^2}$$

EXAMPLE 7 *Calculate mean, variance and standard deviation for the following frequency distribution:*

Class	0–30	30–60	60–90	90–120	120–150	150–180	180–210
Frequency	2	3	5	10	3	5	2

SOLUTION Here $h = 30$. Let the assumed mean be $A = 105$.
Now, we prepare the table given below.

Class	Frequency f_i	Midpoint x_i	$y_i = \frac{(x_i - 105)}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0–30	2	15	–3	9	–6	18
30–60	3	45	–2	4	–6	12
60–90	5	75	–1	1	–5	5
90–120	10	105 = A	0	0	0	0
120–150	3	135	1	1	3	3
150–180	5	165	2	4	10	20
180–210	2	195	3	9	6	18
Total	$N = 30$				2	76

$\therefore \quad A = 105,\ h = 30,\ N = \Sigma f_i = 30,\ \Sigma f_i y_i = 2$ and $\Sigma f_i y_i^2 = 76.$

$\therefore \quad \bar{x} = \left(A + \dfrac{\Sigma f_i y_i}{N} \times h \right) \Rightarrow \bar{x} = \left(105 + \dfrac{2}{30} \times 30 \right) = (105 + 2) = 107.$

Thus, mean = 107.

Variance, $\sigma^2 = \dfrac{h^2}{N^2} \cdot \{N \cdot \Sigma f_i y_i^2 - (\Sigma f_i y_i)^2\}$

$= \dfrac{(30)^2}{(30)^2} \cdot \{30 \times 76 - (2)^2\} = (2280 - 4) = 2276.$

Standard deviation, $\sigma = \sqrt{2276} = 47.71.$

EXAMPLE 8 *Given below are the diameters of circles (in mm) drawn in a design.*

Diameter	33–36	37–40	41–44	45–48	49–52
Number of circles	15	17	21	22	25

Calculate the mean diameter of the circles, variance and standard deviation.

SOLUTION Converting the given series into an exclusive series, we prepare the table given below.

Class	Frequency f_i	Midpoint x_i	$y_i = \dfrac{(x_i - 42.5)}{4}$	y_i^2	$f_i y_i$	$f_i y_i^2$
32.5–36.5	15	34.5	–2	4	–30	60
36.5–40.5	17	38.5	–1	1	–17	17
40.5–44.5	21	42.5 = A	0	0	0	0
44.5–48.5	22	46.5	1	1	22	22
48.5–52.5	25	50.5	2	4	50	100
	$N = 100$				25	199

$\therefore \quad A = 42.5,\ h = 4,\ N = \Sigma f_i = 100,\ \Sigma f_i y_i = 25$ and $\Sigma f_i y_i^2 = 199.$

$\therefore \quad \bar{x} = \left(A + \dfrac{\Sigma f_i y_i}{N} \times h \right) \Rightarrow \bar{x} = \left(42.5 + \dfrac{25}{100} \times 4 \right) = 43.5.$

Thus, mean = 43.5 mm.

Variance, $\sigma^2 = \dfrac{h^2}{N^2} \cdot \{N \cdot \Sigma f_i y_i^2 - (\Sigma f_i\ y_i)^2\}$

$= \dfrac{16}{10000} \times \{100 \times 199 - (25)^2\} = \dfrac{16}{10000} \times (19900 - 625)$

$= \left(\dfrac{16}{10000} \times 19275 \right) = \dfrac{3084}{100} = 30.84.$

Standard deviation, $\sigma = \sqrt{30.84} = 5.55.$

Hence, mean = 43.5 mm, variance = 30.84, and standard deviation = 5.55.

EXERCISE 30B

1. Find the mean, variance and standard deviation for the numbers 4, 6, 10, 12, 7, 8, 13, 12.
2. Find the mean, variance and standard deviation for first six odd natural numbers.

Using short cut method, find the mean, variance and standard deviation for the data:

3.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

4.

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

5.

x_i	10	15	18	20	25
f_i	3	2	5	8	2

6.

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

7.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	8	15	16	6

8.

Class	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Frequency	3	7	12	15	8	3	2

9.

Class	25–35	35–45	45–55	55–65	65–75
Frequency	64	132	153	140	51

ANSWERS (EXERCISE 30B)

1. Mean = 9, Variance = 9.25 and SD = 3.04
2. Mean = 6, Variance = 11.67 and SD 3.41
3. Mean = 14, Variance = 45.8 and SD = 6.77
4. Mean = 19, Variance = 43.4 and SD = 6.59
5. Mean = 18, Variance = 17 and SD = 4.12
6. Mean = 100, Variance = 29.09 and SD = 5.39
7. Mean = 27, Variance = 132 and SD = 11.49
8. Mean = 62, Variance = 201 and SD 14.17
9. Mean = 49.67, Variance = 135.44 and SD = 11.64

MISCELLANEOUS WORD PROBLEMS

EXAMPLE 1 *The mean and variance of seven observations are* 8 *and* 16 *respectively. If five of these are* 2, 4, 10, 12, 14, *find the remaining two observations.*

SOLUTION Let the remaining two observations be x and y. Then,

$$\text{mean} = 8 \Rightarrow \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 42 + x + y = 56$$

$$\Rightarrow x + y = 14. \quad \ldots (i)$$

Also, variance = 16

$$\Rightarrow \frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - 8^2 = 16$$

$$\left[\because \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2\right]$$

$$\Rightarrow \frac{1}{7}(460 + x^2 + y^2) = 80$$

$$\Rightarrow 460 + x^2 + y^2 = 560$$

$$\Rightarrow x^2 + y^2 = 100. \quad \ldots (ii)$$

Now, $(x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$

$$\Rightarrow (x-y)^2 = 2(x^2 + y^2) - (x+y)^2$$

$$\Rightarrow (x-y)^2 = (2 \times 100) - (14)^2 = (200 - 196) = 4$$

$$\Rightarrow x - y = \pm 2.$$

Now, $x + y = 14$, $x - y = 2 \Rightarrow x = 8, y = 6$;

$x + y = 14$, $x - y = -2 \Rightarrow x = 6, y = 8$.

Hence, the remaining two observations are 6 and 8.

EXAMPLE 2 *The mean and variance of six observations are* 8 *and* 16 *respectively. If each observation is multiplied by* 3, *find the new mean and new variance of the resulting observations.*

SOLUTION Let the given observations be $x_1, x_2, x_3, x_4, x_5, x_6$.

Then, mean = 8 $\Rightarrow \frac{1}{6}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = 8$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 48. \quad \ldots (i)$$

Also, variance = 16

$$\Rightarrow \frac{1}{6}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) - 8^2 = 16 \quad \left[\because \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2\right]$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 480. \quad \ldots (ii)$$

When each observation is multiplied by 3, then new observations are $3x_1, 3x_2, 3x_3, 3x_4, 3x_5$ and $3x_6$.

$$\therefore \quad \text{new mean} = \frac{1}{6}(3x_1 + 3x_2 + 3x_3 + 3x_4 + 3x_5 + 3x_6)$$

$$= \frac{3}{6}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = \left(\frac{1}{2} \times 48\right) = 24 \text{ [using (i)]}$$

$$\therefore \quad \text{new variance} = \frac{(3x_1)^2 + (3x_2)^2 + (3x_3)^2 + (3x_4)^2 + (3x_5)^2 + (3x_6)^2}{6} - (24)^2$$

$$= \frac{9}{6}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) - 576$$

$$= \left(\frac{9}{6} \times 480\right) - 576 = (720 - 576) = 144 \qquad \text{[using (ii)].}$$

Hence, new mean = 24 and new variance = 144.

EXAMPLE 3 *The mean and standard deviation of* 20 *observations are found to be* 10 *and* 2 *respectively. On rechecking it was found that an observation* 8 *was incorrect. Calculate the correct mean and standard deviation when*

(i) the wrong item is omitted,

(ii) the wrong item is replaced by 12.

SOLUTION $\text{Mean} = 10 \Rightarrow \dfrac{\sum_{i=1}^{20} x_i}{20} = 10$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200.$$

$$\text{SD} = 2 \Rightarrow \sigma^2 = 4$$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4$$

$$\Rightarrow \sum_{i=1}^{20} x_i^2 = 2080.$$

Thus, incorrect $\left(\sum_{i=1}^{20} x_i^2\right) = 200$ and incorrect $\left(\sum_{i=1}^{20} x_i^2\right) = 2080.$

CASE (i) *When the wrong item is omitted*

On omitting 8, we are left with 19 observations.

$$\therefore \quad \text{correct}\left(\sum_{i=1}^{19} x_i\right) = \text{incorrect}\left(\sum_{i=1}^{20} x_i\right) - 8$$

$$= (200 - 8) = 192.$$

Thus, correct $\left(\sum_{i=1}^{19} x_i\right) = 192$

$$\therefore \quad \text{correct mean} = \frac{192}{19} = 10.105. \qquad \text{... (i)}$$

$$\text{Also, correct}\left(\sum_{i=1}^{19} x_i^2\right) = \text{incorrect}\left(\sum_{i=1}^{20} x_i^2\right) - 64$$

$$= (2080 - 64) = 2016.$$

$$\therefore \quad \text{correct variance} = \frac{1}{19}\left(\text{correct}\sum_{i=1}^{19} x_i^2\right) - (\text{correct mean})^2$$

$$= \left(\frac{1}{19} \times 2016\right) - \left(\frac{192}{19}\right)^2 = \left(\frac{2016}{19} - \frac{36864}{361}\right)$$

$$= \frac{(38304 - 36864)}{361} = \frac{1440}{361}.$$

$$\therefore \quad \text{correct SD} = \sqrt{\frac{1440}{361}} = \frac{12\sqrt{10}}{19} = \frac{(12 \times 3.162)}{19}$$

$$= \frac{37.944}{19} = 1.997.$$

$$\therefore \quad \text{mean} = 10.105 \quad \text{and} \quad \text{SD} = 1.997.$$

CASE (ii) *When incorrect observation* 8 *is replaced by* 12.

$$\text{Incorrect}\left(\sum_{i=1}^{20} x_i\right) = 200$$

$$\Rightarrow \quad \text{correct}\left(\sum_{i=1}^{20} x_i\right) = (200 - 8 + 12) = 204$$

$$\Rightarrow \quad \text{correct mean} = \frac{204}{20} = 10.2.$$

$$\text{Also, incorrect}\left(\sum_{i=1}^{20} x_i^2\right) = 2080$$

$$\Rightarrow \quad \text{correct}\left(\sum_{i=1}^{20} x_i^2\right) = (2080 - 8^2 + 12^2) = 2160$$

$$\Rightarrow \quad \text{correct variance} = \frac{\text{correct}\left(\sum_{i=1}^{20} x_i^2\right)}{20} - (\text{correct mean})^2$$

$$= \left\{\frac{2160}{20} - (10.2)^2\right\} = (108 - 104.04) = 3.96.$$

$$\Rightarrow \quad \text{correct SD} = \sqrt{3.96} = 1.989$$

Thus, in this case, we have mean = 10.2 and SD = 1.989.

EXAMPLE 4 *The mean and standard deviation of* 100 *observations were calculated as* 40 *and* 5.1 *respectively by a student who took by mistake* 50 *instead of* 40 *for one observation. What are the correct mean and standard deviation?*

SOLUTION $\text{Mean} = 40 \Rightarrow \dfrac{\sum_{i=1}^{100} x_i}{100} = 40$

$$\Rightarrow \sum_{i=1}^{100} x_i = 4000.$$

$$\text{SD} = 5.1 \Rightarrow \sigma^2 = (5.1)^2$$

$$\Rightarrow \frac{\sum_{i=1}^{100} x_i^2}{100} - (40)^2 = 26.01$$

$$\Rightarrow \sum_{i=1}^{100} x_i^2 = 162601.$$

$$\text{Thus, incorrect} \left(\sum_{i=1}^{100} x_i \right) = 4000 \text{ and incorrect} \left(\sum_{i=1}^{100} x_i^2 \right) = 162601$$

$$\text{Now, incorrect} \left(\sum_{i=1}^{100} x_i \right) = 4000$$

$$\Rightarrow \text{correct} \left(\sum_{i=1}^{100} x_i \right) = (4000 - 50 + 40) = 3990$$

$$\Rightarrow \text{correct mean} = \frac{3990}{100} = 39.9. \qquad \text{... (i)}$$

$$\text{And, incorrect} \left(\sum_{i=1}^{100} x_i^2 \right) = 162601$$

$$\Rightarrow \text{correct} \left(\sum_{i=1}^{100} x_i^2 \right) = \{162601 - (50)^2 + (40)^2\} = 161701$$

$$\Rightarrow \text{correct variance} = \frac{\text{correct} \left(\sum_{i=1}^{100} x_i^2 \right)}{100} - (\text{correct mean})^2$$

$$= \left\{ \frac{161701}{100} - (39.9)^2 \right\} = \{1617.01 - (40 - 0.1)^2\}$$

$$= (1617.01) - \{1600 + 0.01 - 8\}$$

$$= (1617.01 - 1592.01) = 25$$

$$\Rightarrow \text{correct SD} = \sqrt{25} = 5.$$

Hence, correct mean = 39.9 and correct SD = 5.

EXAMPLE 5 *If each of the observations $x_1, x_2, x_3, \ldots, x_n$ is increased by an amount a, where a is a negative or positive number, then show that the variance remains unchanged.*

SOLUTION Let $\bar{x}$ be the mean of $x_1, x_2, x_3, \ldots, x_n$. Then,

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n).$$

Let $y_i = x_i + a$ for each $i = 1, 2, 3, \ldots, n$.

Let $\bar{y}$ be the mean of $y_1, y_2, y_3, \ldots, y_n$. Then,

$$\bar{y} = \frac{1}{n}(y_1 + y_2 + y_3 + \ldots + y_n)$$

$$= \frac{1}{n}(x_1 + a + x_2 + a + x_3 + a + \ldots + x_n + a)$$

$$= \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n) + \frac{1}{n}(a + a + a + \ldots n \text{ times})$$

$$= \bar{x} + \frac{1}{n}(na) = (\bar{x} + a).$$

$\therefore \quad \bar{y} = (\bar{x} + a).$

Now, the variance of the new observations is given by

$$\text{variance}\,(y) = \sigma^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \{(x_i + a) - (\bar{x} + a)\}^2$$

$$[\because \quad y_i = (x_i + a) \quad \text{and} \quad \bar{y} = (\bar{x} + a)]$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i + a - \bar{x} - a)^2$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \text{variance}\,(x).$$

Hence, the variance of the new observations is the same as the variance of the original observations.

EXAMPLE 6 *If the mean and variance of the observations $x_1, x_2, x_3, \ldots, x_n$ are $\bar{x}$ and σ^2 respectively and a be a nonzero real number, then show that the mean and variance of $ax_1, ax_2, ax_3, \ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$ respectively.*

SOLUTION Let $\bar{x}$ be the mean of $x_1, x_2, x_3, \ldots, x_n$ and a be a nonzero real number.

Then, $\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n).$

Let $y_i = ax_i$ for each $i = 1, 2, 3, \ldots, n$. Then,

$$\bar{y} = \frac{1}{n}(y_1 + y_2 + y_3 + \ldots + y_n)$$

$$= \frac{1}{n}(ax_1 + ax_2 + ax_3 + \ldots + ax_n) = a \cdot \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n) = a\bar{x}.$$

Thus, $\bar{y} = a\bar{x}$.

Now, the variance of new observations is given by

$$\text{variance}\,(y) = \sigma_1^2$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} (ax_i - a\bar{x})^2 \quad [\because \quad y_i = ax_i \text{ for each } i \text{ and } \bar{y} = a\bar{x}\,]$$

$$= a^2 \cdot \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= a^2 \cdot \{\text{variance}\,(x)\} = a^2\sigma^2.$$

$\therefore$ new variance $= a^2\sigma^2$.

REMARK $\sigma_1 = \sqrt{a^2\sigma^2} = |a| \cdot \sigma$.

EXERCISE 30C

1. If the standard deviations of the numbers 2, 3, $2x$, 11 is 3.5, calculate the possible values of x.

2. The variance of 15 observations is 6. If each observation is increased by 8, find the variance of the resulting observations.

3. The variance of 20 observations is 5. If each observation is multiplied by 2, find the variance of the resulting observations.

4. The mean and variance of five observations are 6 and 4 respectively. If three of these are 5, 7 and 9, find the other two observations.

5. The mean and variance of five observations are 4.4 and 8.24 respectively. If three of these are 1, 2 and 6, find the other two observations.

6. The mean and standard deviation of 18 observations are found to be 7 and 4 respectively. On rechecking it was found that an observation 12 was misread as 21. Calculate the correct mean and standard deviation.

7. For a group of 200 candidates, the mean and standard deviations of scores were found to be 40 and 15 respectively. Later on it was discovered that the score of 43 was misread as 34. Find the correct mean and standard deviation.

8. The mean and standard deviations of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observations 21, 12 and 18 were incorrect. Find the mean and standard deviation if the incorrect observations were omitted.

ANSWERS (EXERCISE 30C)

1. 3 and $\frac{7}{3}$	**2.** 6	**3.** 20	**4.** 3 and 6
5. 4 and 9	**6.** 6.5, 2.5	**7.** 40.045, 14.995	**8.** 20.06, 14.4

HINTS TO SOME SELECTED QUESTIONS

1. $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2 \Rightarrow 3x^2 - 16x + 21 = 0.$

COEFFICIENT OF VARIATION

COEFFICIENT OF VARIATION

Let the mean and standard deviation of the given data be $\bar{x}$ and σ respectively. Then, the coefficient of variation is defined as

$$\text{CV} = \left(\frac{\sigma}{\bar{x}} \times 100\right), \text{ where } \bar{x} \neq 0.$$

For comparing the dispersion of two series, we calculate the coefficient of variation for each series.

The series whose CV is more, is more variable than the other.

COMPARISON OF TWO FREQUENCY DISTRIBUTIONS WITH SAME MEAN

Let there be two frequency distributions with the same mean $\bar{x}$ and having standard deviations σ_1 and σ_2 respectively.

Then, CV for 1st distribution $= \left(\frac{\sigma_1}{\bar{x}} \times 100\right)\%$.

And, CV for 2nd distribution $= \left(\frac{\sigma_2}{\bar{x}} \times 100\right)\%$,

Thus, $\frac{\text{CV}_1}{\text{CV}_2} = \frac{\sigma_1}{\sigma_2}$, where CV_1 and CV_2 denote the CV for 1st and 2nd distributions respectively.

Thus, *in two series having the same mean, the one having more standard deviation is more variable.*

EXAMPLE 1 *The following results show the number of workers and the wages paid to them in two factories A and B.*

Factory	*A*	*B*
Number of workers	4000	5000
Mean wages	Rs 3500	Rs 3500
Variance of distribution of wages	64	81

Which factory has more variation in wages?

SOLUTION We have:

Variance of distribution of wages in factory A, $\sigma_1^2 = 64$.

Variance of distribution of wages in factory B, $\sigma_2^2 = 81$.

$\therefore \quad (\sigma_1^2 = 64 \quad \text{and} \quad \sigma_2^2 = 81) \Rightarrow (\sigma_1 = 8 \quad \text{and} \quad \sigma_2 = 9)$.

Thus, SD of distribution of wages in factory $A = 8$.

And, SD of distribution of wages in factory $B = 9$.

Since the monthly mean wages in two factories are the same and $\sigma_1 < \sigma_2$, hence, the factory *B* has more variation in wages.

EXAMPLE 2 *Coefficient of variation of two distributions are* 60% *and* 75%, *and their standard deviations are* 18 *and* 15 *respectively. Find their arithmetic means.*

SOLUTION Given: $CV_1 = 60$, $CV_2 = 75$, $\sigma_1 = 18$ and $\sigma_2 = 15$.
Let $\bar{x}_1$ and $\bar{x}_2$ be the means of 1st and 2nd distribution respectively.
Then,

$$CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow \bar{x}_1 = \frac{\sigma_1 \times 100}{CV_1}$$

$$\Rightarrow \bar{x}_1 = \frac{18 \times 100}{60} = 30.$$

$$CV_2 = \frac{\sigma_2}{x_2} \times 100 \Rightarrow \bar{x}_2 = \frac{\sigma_2 \times 100}{CV_2}$$

$$\Rightarrow \bar{x}_2 = \frac{15 \times 100}{75} = 20.$$

Hence, $\bar{x}_1 = 30$ and $\bar{x}_2 = 20$.

EXAMPLE 3 *The mean and variance of the heights and weights of the students of a class are given below.*

	Height	*Weight*
Mean	160 cm	50.4 kg
Variance	116.64 cm^2	17.64 kg^2

Show that the weights are more variable than heights.

SOLUTION *For height, we have:*
Variance, $\sigma^2 = 116.64\ cm^2 \Rightarrow$ SD, $\sigma = \sqrt{116.64}$ cm = 10.8 cm.
Also, mean height = 160 cm.

$$\therefore \quad CV_1 = \left(\frac{SD}{mean} \times 100\right)\% = \left(\frac{10.8}{160} \times 100\right)\% = 4.75\%$$

For weight, we have:
Variance, $\sigma^2 = 17.64\ kg^2 \Rightarrow$ SD, $\sigma = \sqrt{17.64}$ kg = 4.2 kg.
And, mean weight = 50.4 kg.

$$\therefore \quad CV_2 = \left(\frac{SD}{mean} \times 100\right)\% = \left(\frac{4.2}{50.4} \times 100\right)\% = \frac{25}{3}\% = 8\frac{1}{3}\%.$$

Clearly, (CV in weights) > (CV in heights).
Hence, weights are more variable than heights.

EXERCISE 30D

1. The following results show the number of workers and the wages paid to them in two factories F_1 and F_2.

Factory	*A*	*B*
Number of workers	3600	3200
Mean wages	Rs 5300	Rs 5300
Variance of distribution of wages	100	81

Which factory has more variation in wages?

2. Coefficient of variation of two distributions are 60% and 80% respectively, and their standard deviations are 21 and 16 respectively. Find their arithmetic means.

3. The mean and variance of the heights and weights of the students of a class are given below:

	Heights	Weights
Mean	63.2 inches	63.2 kg
SD	11.5 inches	5.6 kg

Which shows more variability, heights or weights?

4. The following results show the number of workers and the wages paid to them in two factories *A* and *B* of the same industry.

Firms	*A*	*B*
Number of workers	560	650
Mean monthly wages	Rs 5460	Rs 5460
Variance of distribution of wages	100	121

(i) Which firm pays larger amount as monthly wages?

(ii) Which firm shows greater variability in individual wages?

5. The sum and the sum of squares of length x (in cm) and weight y (in g) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8,\ \sum_{i=1}^{50} y_i = 261 \quad \text{and} \quad \sum_{i=1}^{50} y_i^2 = 1457.6.$$

Which is more variable, the length or weight?

ANSWERS (EXERCISE 30D)

1. A **2.** 35, 20 **3.** Heights **4.** (i) A (ii) B

5. Weight

HINTS TO SOME SELECTED QUESTIONS

5. $\text{var}(x) = \frac{1}{50}\cdot\sum_{i=1}^{50} x_i^2 - \left(\frac{1}{50}\cdot\sum_{i=1}^{50} x_i\right)^2 = \left(\frac{902.8}{50}\right) - \left(\frac{212}{50}\right)^2 = \frac{451.4}{25} - \left(\frac{106}{25}\right)^2$

$= \left(\frac{451.4}{25} - \frac{11236}{625}\right) = \left(\frac{11285 - 11236}{625}\right)$

$= \frac{49}{625} = 0.078.$

$\text{var}(y) = \frac{1}{50}\cdot\sum_{i=1}^{50} y_i^2 - \left(\frac{1}{50}\cdot\sum_{i=1}^{50} y_i\right)^2 = \left\{\frac{1457.6}{50} - \left(\frac{261}{50}\right)^2\right\}$

$= \{29.152 - (5.22)^2\} = 29.152 - 27.2484$

$= 1.9036.$

$\therefore$ $\text{var}(x) < \text{var}(y)$.